Dynamics of **viscous** liquid film and ligament stretching

Benoit Scheid

TIPs – Transferts, Interfaces et Procédés
Université Libre de Bruxelles

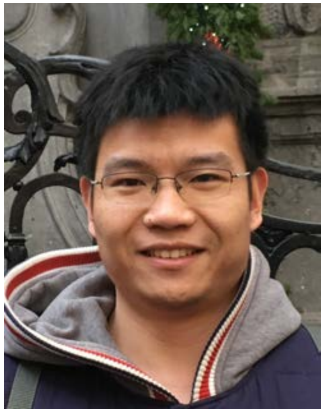
GDR TransINTER, Marcgh 17th 2021

Lorène Champougny

Emmanuelle Rio

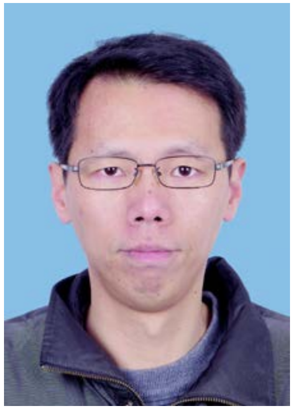
Frédéric Restagno

"The break-up of free films pulled out of a pure liquid bath"
Journal of Fluid Mechanics **811**, 499-524 (2017)



Wei Xiaofeng

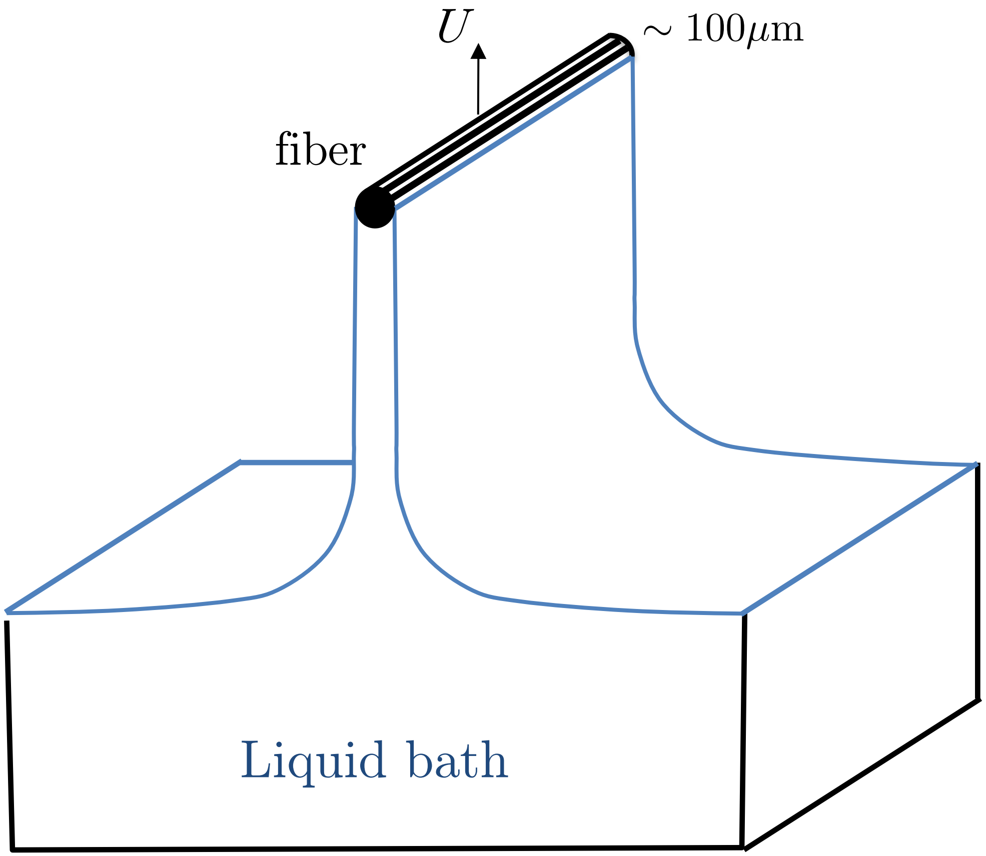
Javier Rivero-Rodriguez



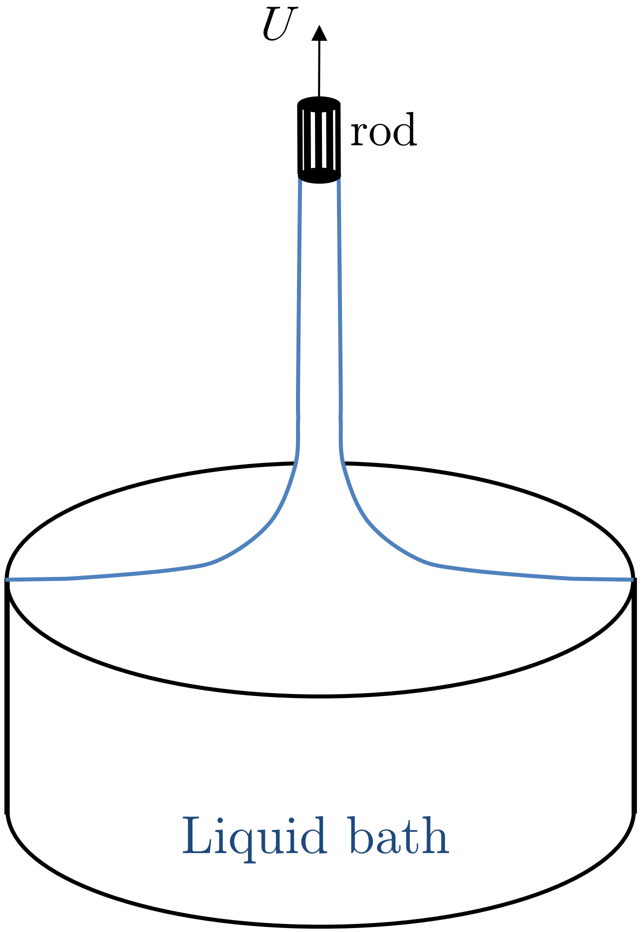
Jun Zou

"Statics and dynamics of a viscous ligament drawn out of a pure-liquid bath"
Under review to Journal of Fluid Mechanics

1) Film

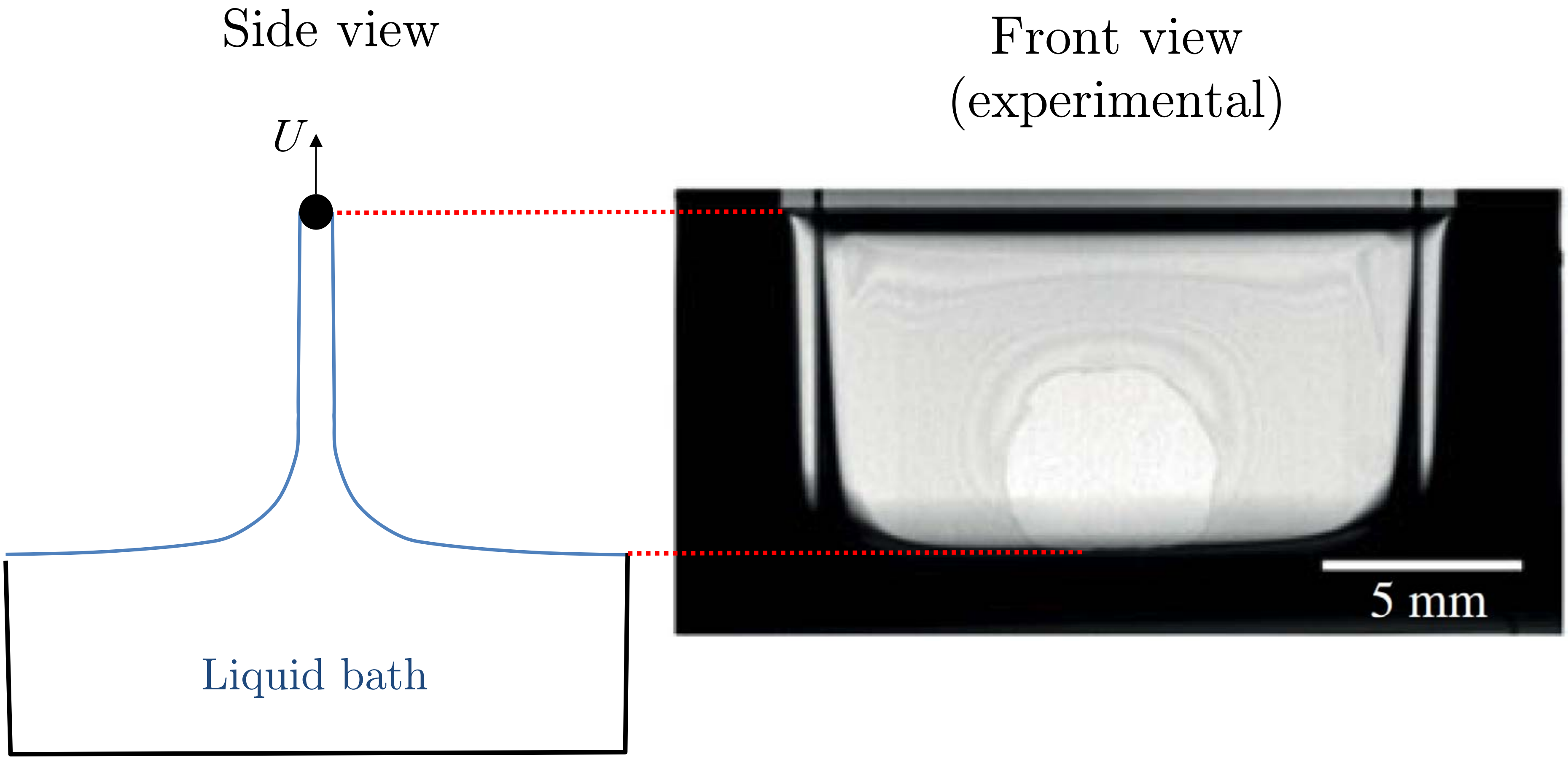


2) Ligament



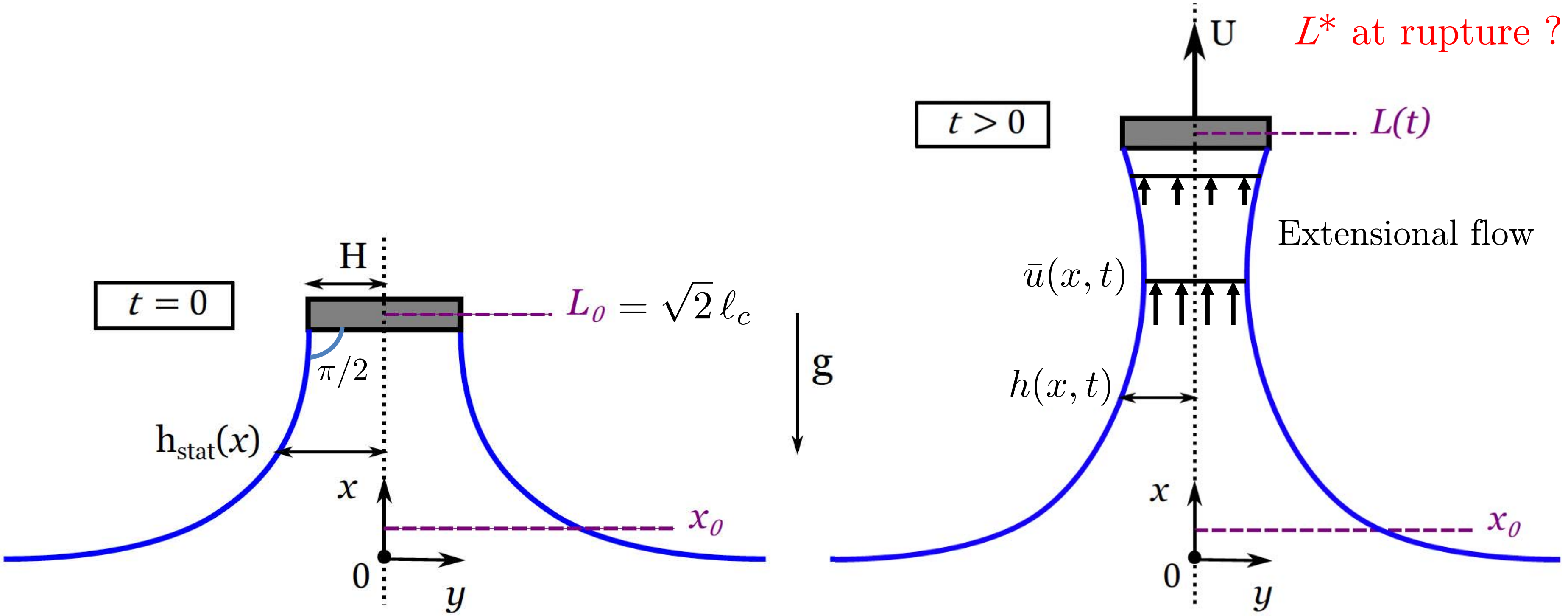
UNSTEADY → Numerical and Experimental Investigations

1) Pulling free-standing films



Break-up mechanism: intermolecular forces (van der Waals)
→ Important when the film thickness < 100 nm

Pulling free-standing films



L^* at rupture ?

Film pulling parameters:

- pulling velocity: U
- fiber half-width: H

$$\boxed{\ell_c = \sqrt{\frac{\gamma}{\rho g}}}$$

Liquid properties:

- density: ρ
- viscosity: η
- surface tension: γ
- Hamaker constant: A_H

Lubrication model for an extensional flow

$$\boxed{\begin{aligned} \partial_t h + \partial_x\left(\bar{u}h\right) = 0, \\ \mathrm{We}\, h\left(\partial_t \bar{u} + \bar{u}\partial_x \bar{u}\right) - h\left(2\varepsilon\partial_x K - 1 + \mathcal{A}\frac{\partial_x h}{h^4}\right) - 4\mathrm{Ca}\,\partial_x\left(h\partial_x \bar{u}\right) = 0, \end{aligned}}$$

$$\varepsilon = \frac{H}{\ell_c} \ll 1 \qquad \mathrm{Ca} = \frac{\eta U}{\gamma}, \qquad \mathrm{We} = \frac{\rho U^2 \ell_c}{\gamma} \qquad \text{and} \qquad \mathcal{A} = \frac{A_H \ell_c}{16\pi\gamma H^3}$$

Film pulling parameters:

- pulling velocity: U
- fiber half-width: H

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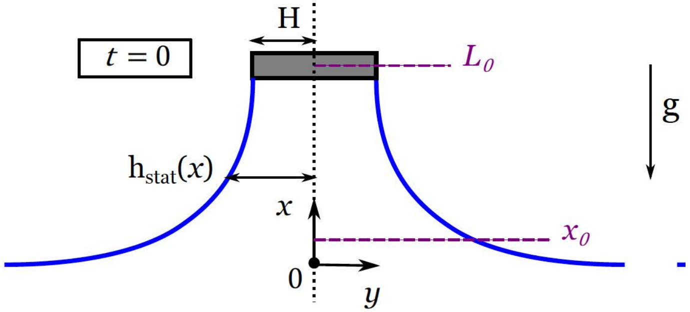
$$K(x,t) = \frac{\partial_{xx} h}{2\left[1 + (\varepsilon \partial_x h)^2\right]^{3/2}}$$

$$h_{\text{stat}}(x) = 1 + \frac{1}{\varepsilon}\left[\sqrt{4 - L_0^2} - \sqrt{4 - x^2} - \operatorname{arctanh}\left(\frac{2}{\sqrt{4 - L_0^2}}\right) + \operatorname{arctanh}\left(\frac{2}{\sqrt{4 - x^2}}\right)\right]$$

$$\begin{cases} \bar{u} = 1 \\ h = 1 \end{cases}$$

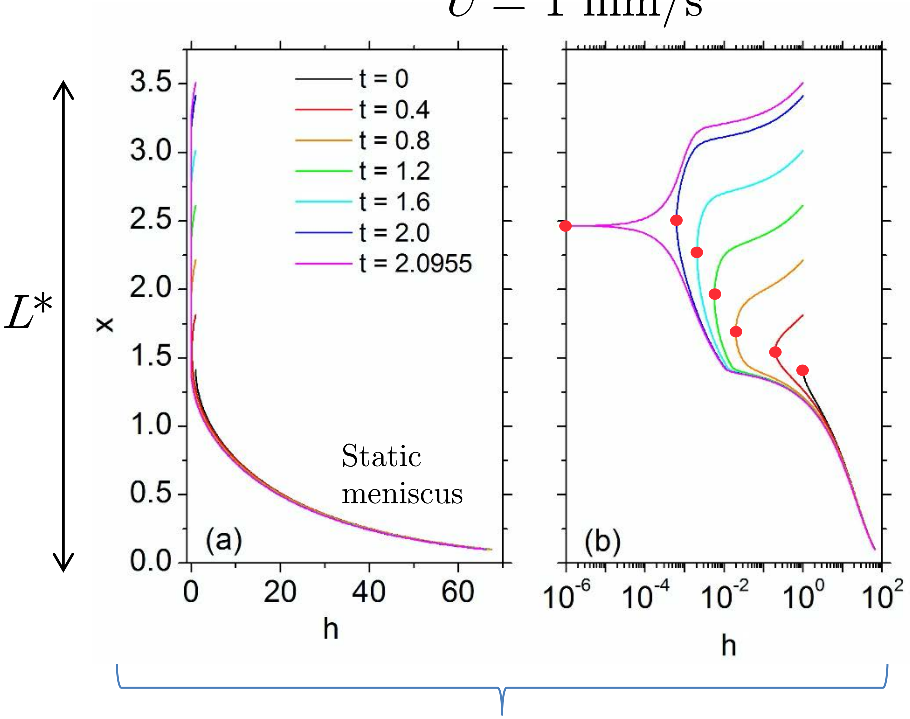


$$\begin{cases} \partial_x h = h'_{\text{stat}} \\ \partial_{xx} h = h''_{\text{stat}} \end{cases}$$

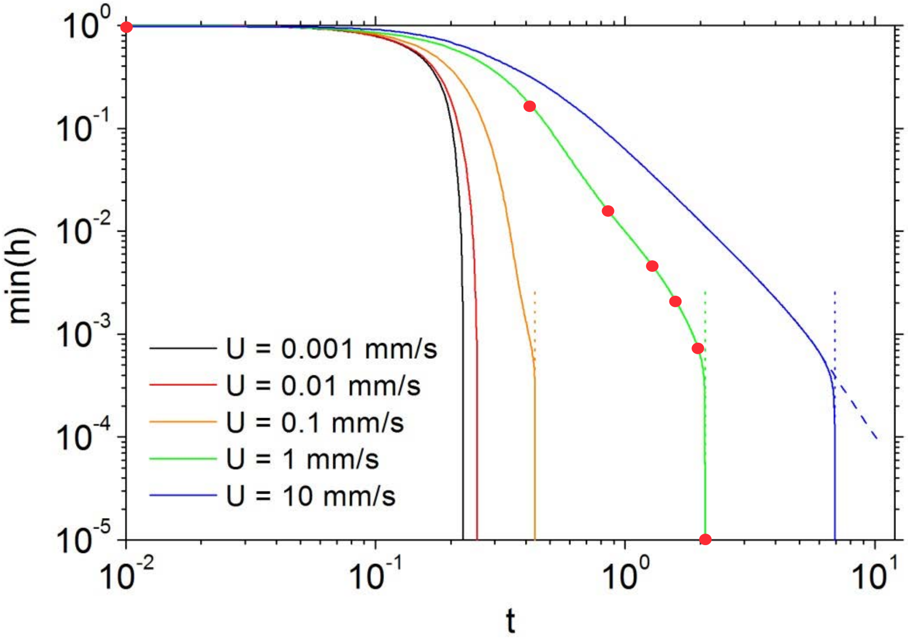
Time-dependent simulations 1D

COMSOL: PDE solver + Moving Mesh (ALE)

$U = 1$ mm/s



Thickness profiles

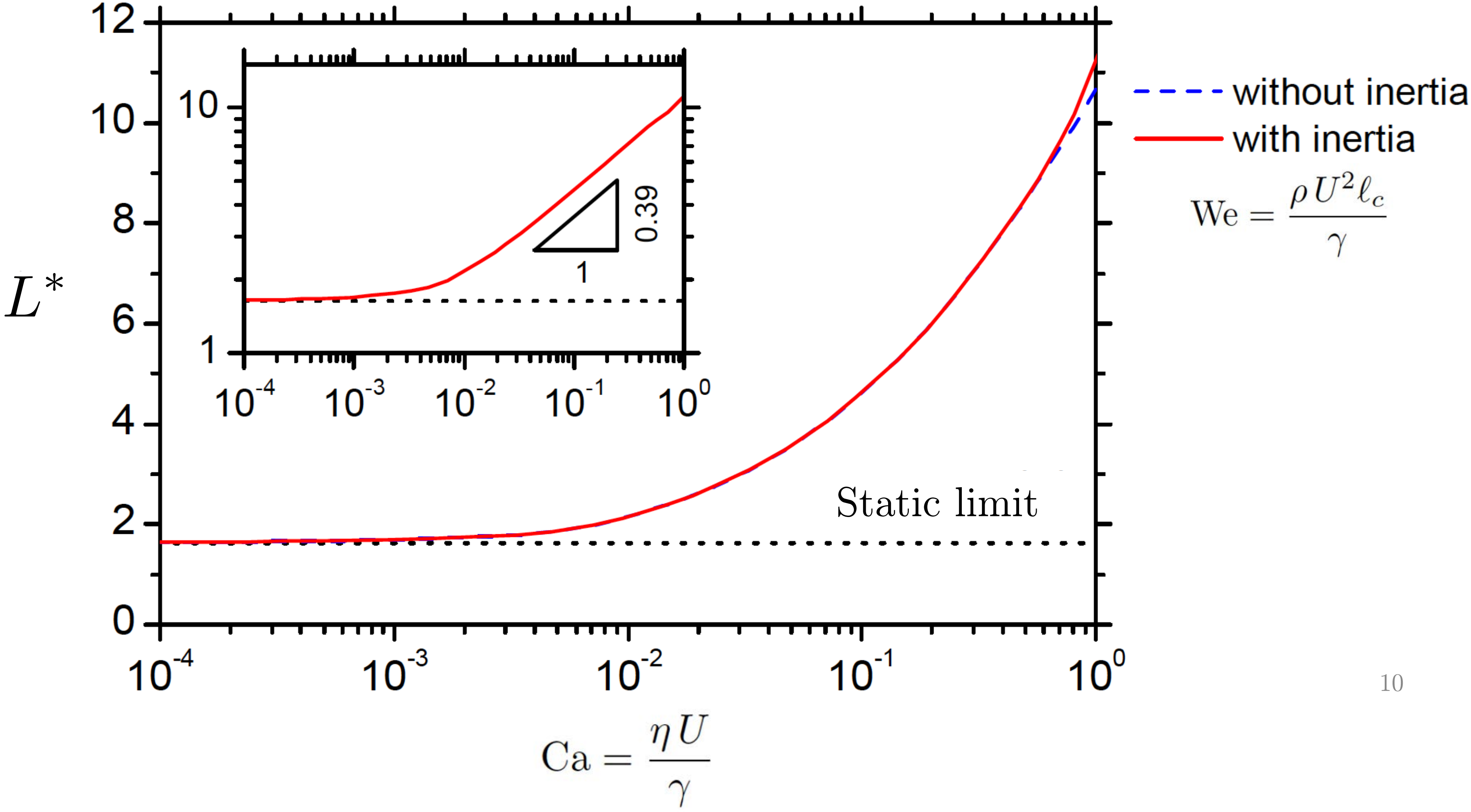


Minimum thickness

→ L^* = film height at break-up

Results of the model: film height at break-up

(Fixed liquid properties)

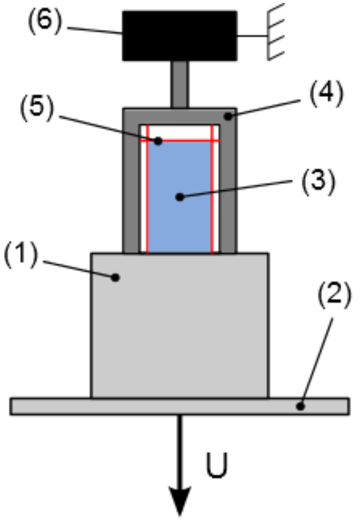


$$\mathrm{We} = \frac{\rho U^2 \ell_c}{\gamma}$$

$$\mathrm{Ca} = \frac{\eta U}{\gamma}$$

Experiments with silicone oil films

Experimental setup



(1) liquid container*

(2) vertical translation plate*

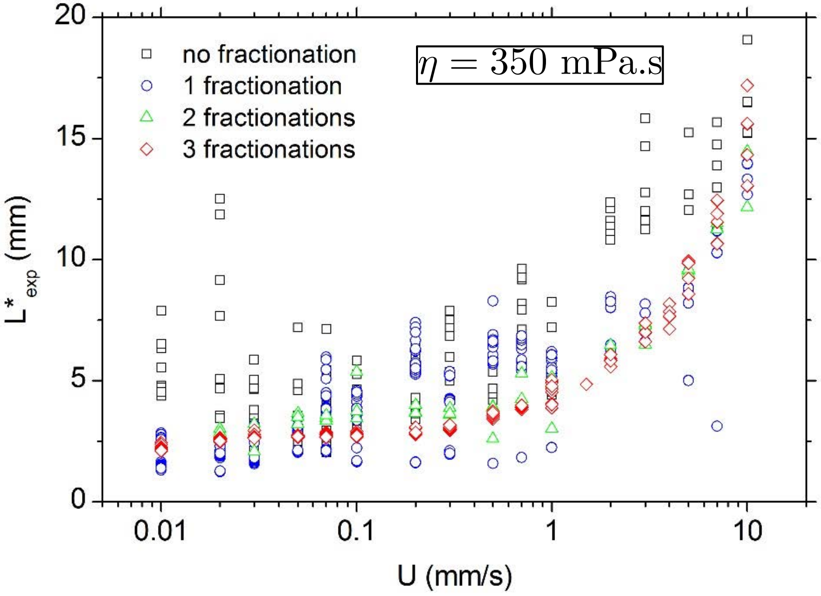
(3) thin liquid film

(4) frame

(5) fishing lines

(6) force transducer

Pure liquid?



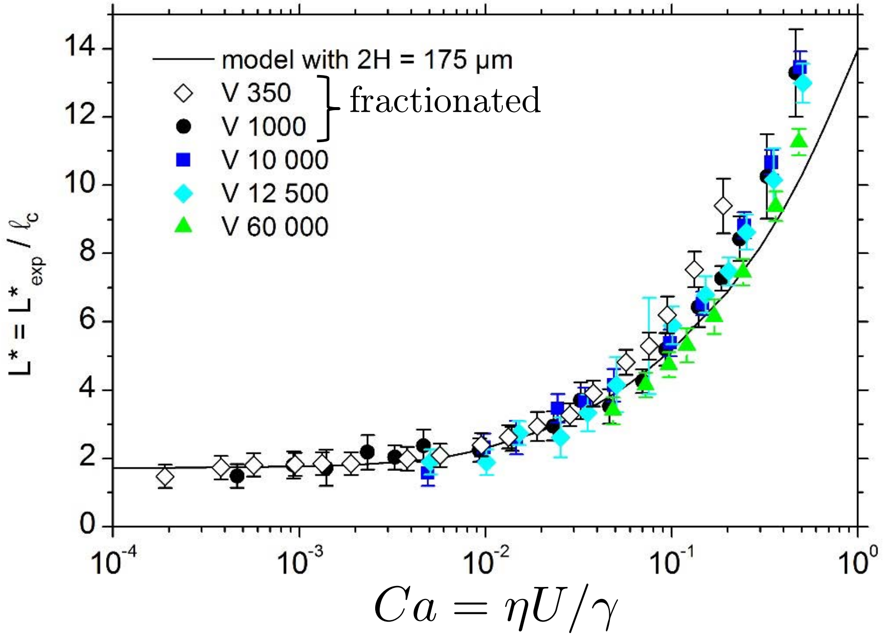
Film pulling parameters:

- pulling velocity: U
- fiber radius: H

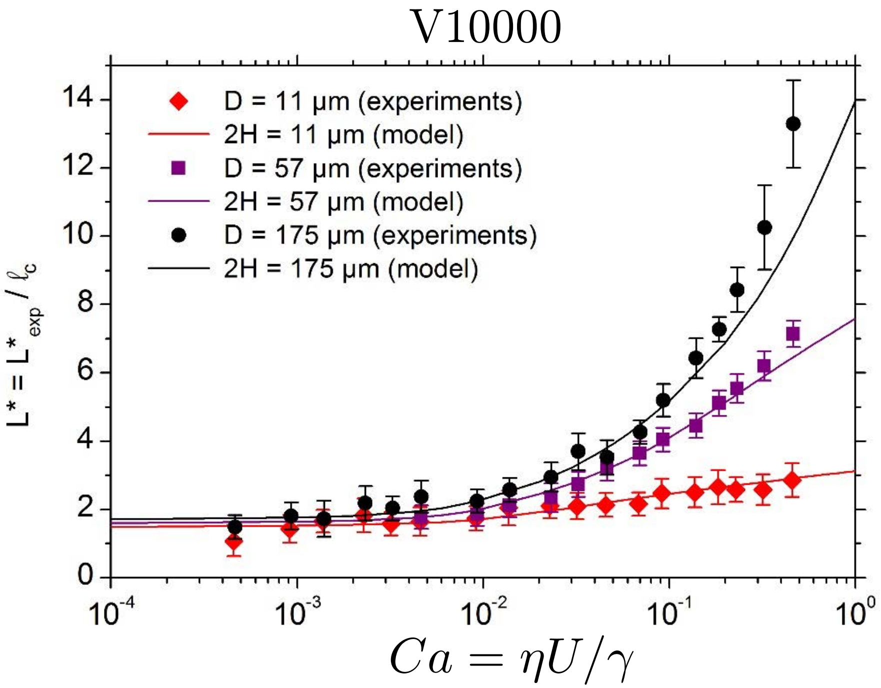
Liquid properties:

- density: ρ
- viscosity: η
- surface tension: γ
- Hamaker constant: A_{H}

Experimental results



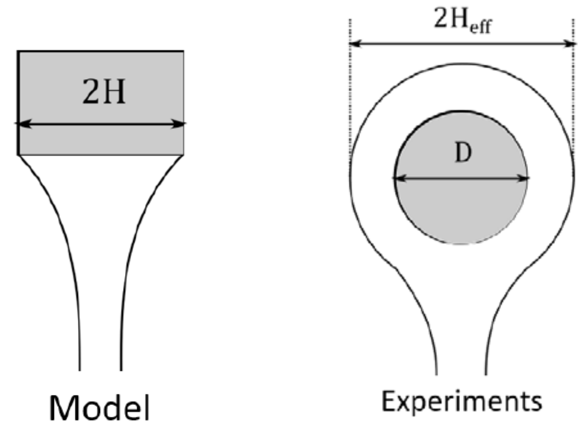
Changing the liquid viscosity



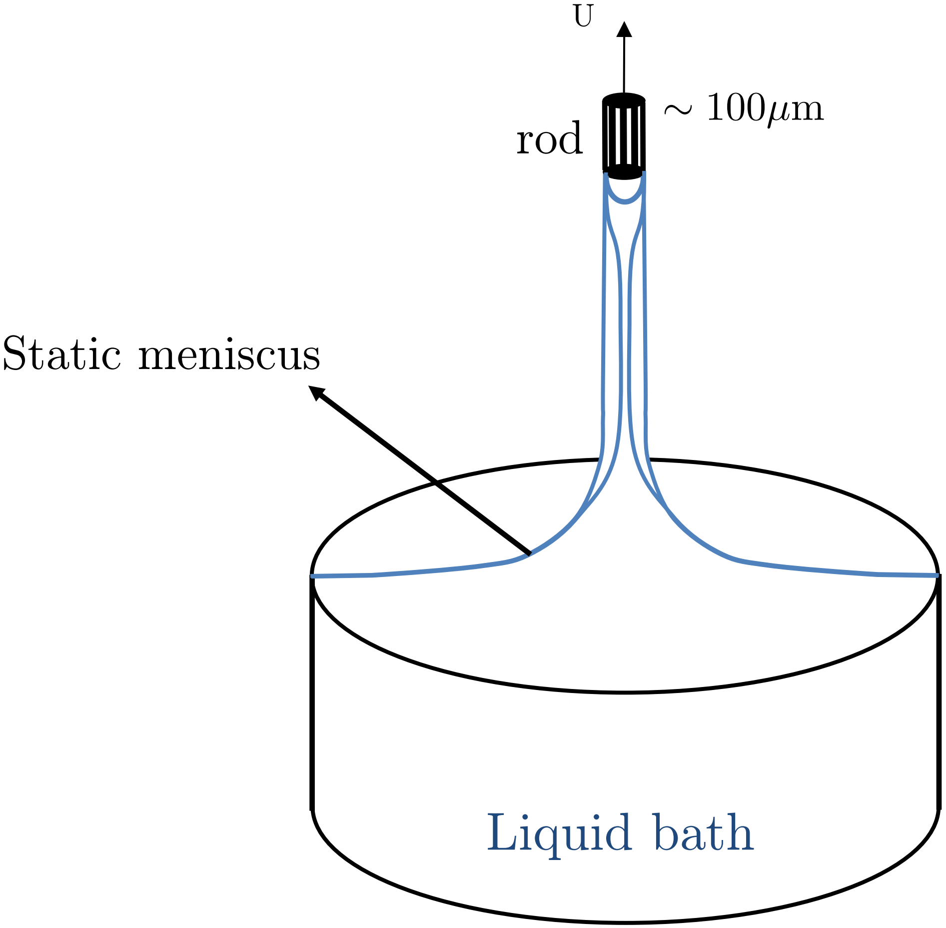
Changing the fiber diameter D

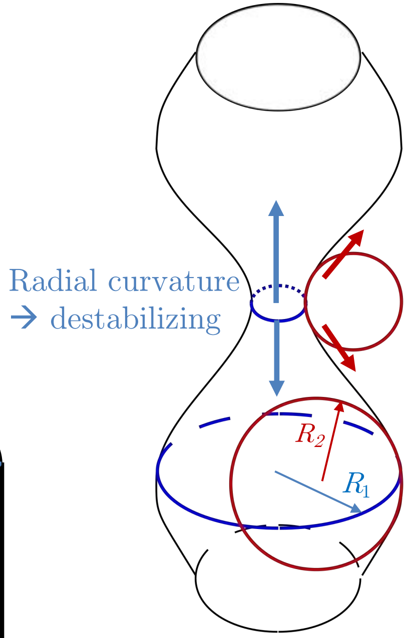
→ Hypothesis to explain the deviation at high capillary numbers: **fiber coating**

→ Small height: max 1 cm for Ca = 1 unless surfactants are added…



2) Ligament





$$\boxed{p_{\mathrm{cap}} = \gamma\left(\frac{1}{R_1} \pm \frac{1}{R_2}\right)}$$

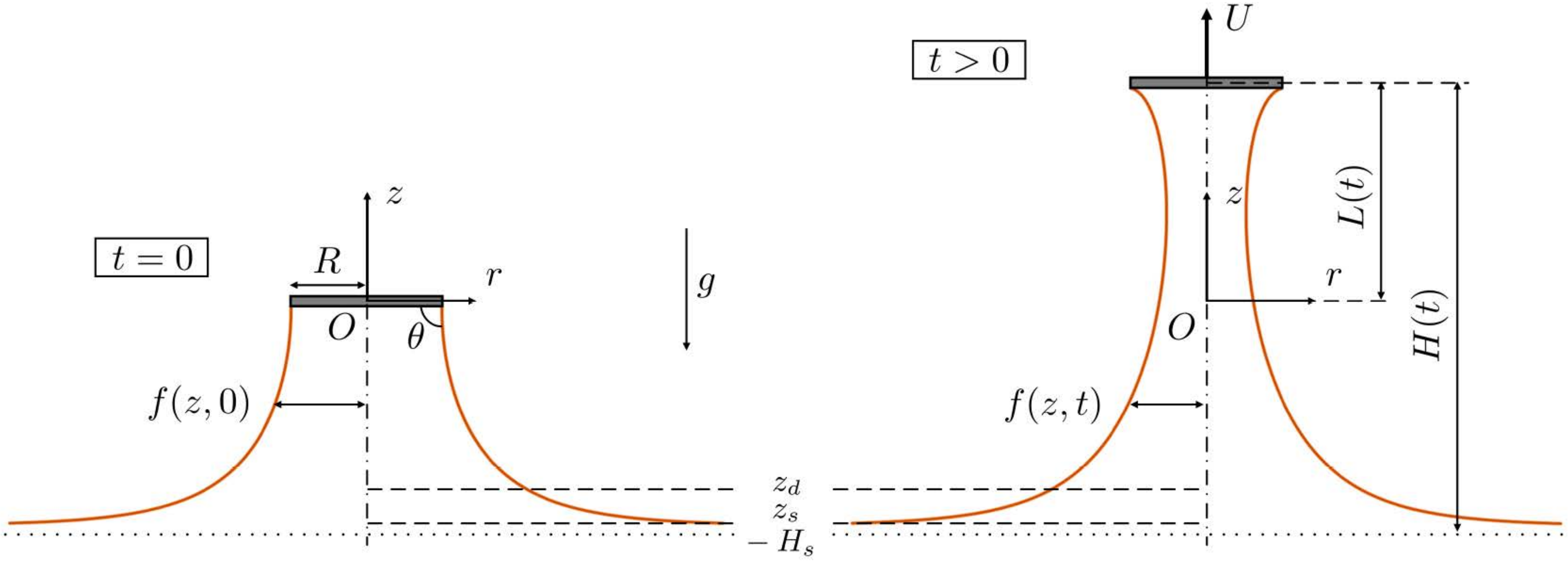
(Laplace pressure)

Break-up mechanism: Rayleigh-Plateau

→ Always present during stretching

Ligament drawing

H_b at rupture ?



$$\boxed{\begin{aligned} \partial_t f^2 + \partial_z (f^2 u) &= 0 \\ \frac{1}{Oh^2} f^2 \left(\partial_t u + u \partial_z u\right) + f^2 \left(\partial_z K + \varepsilon^2\right) - 3\partial_z \left(f^2 \partial_z u\right) &= 0 \end{aligned}}$$

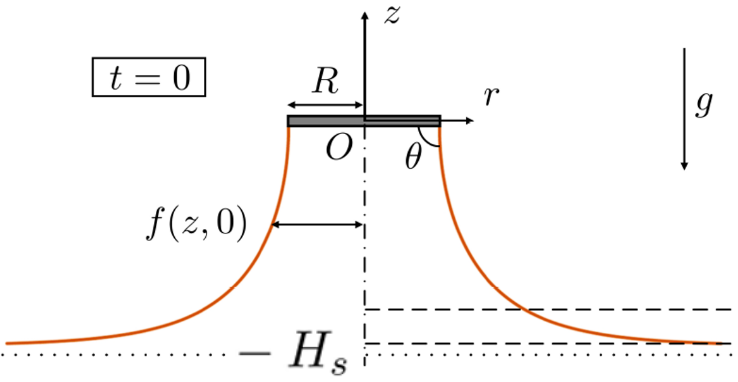
$$\varepsilon = \frac{R}{\ell_c} << 1$$

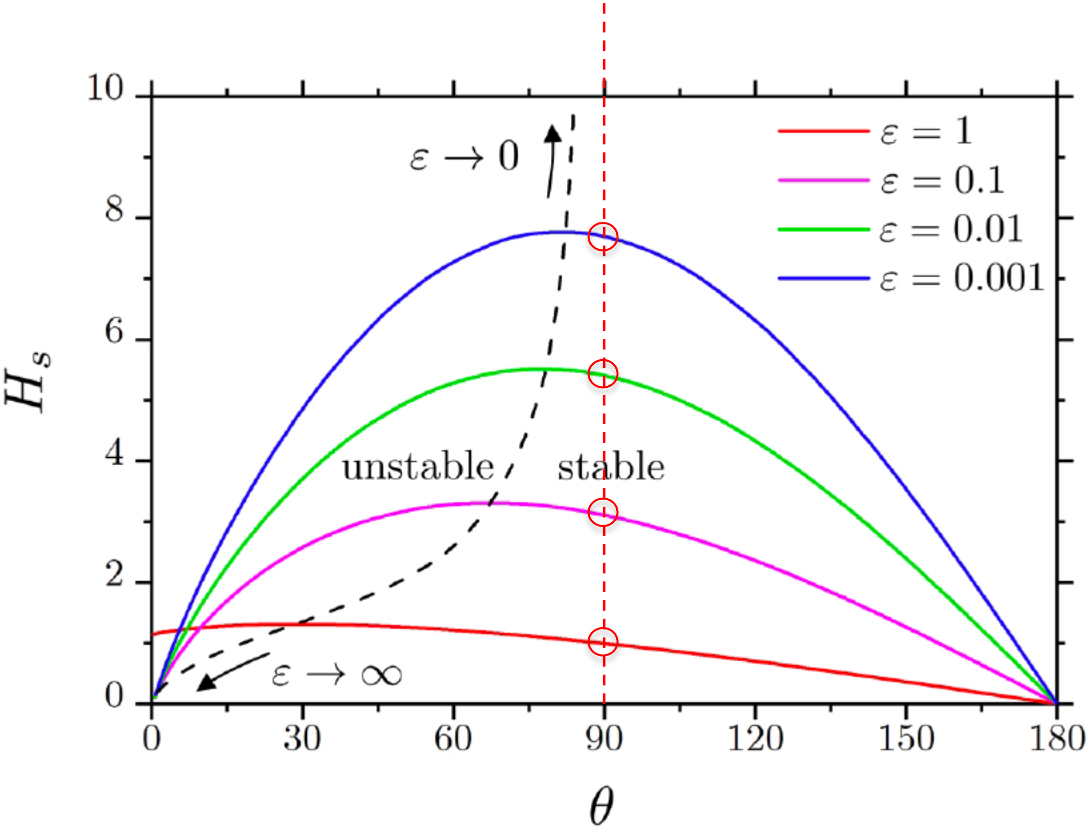
$$K(z,t) = \frac{1}{f\left[1 + (\partial_z f)^2\right]^{1/2}} - \frac{\partial_{zz} f}{\left[1 + (\partial_z f)^2\right]^{3/2}}$$

$$Oh = \frac{\mu}{\sqrt{\rho \gamma \ell_c \varepsilon}}$$

Ligament drawing
Quasi-static approach

No analytical solution!





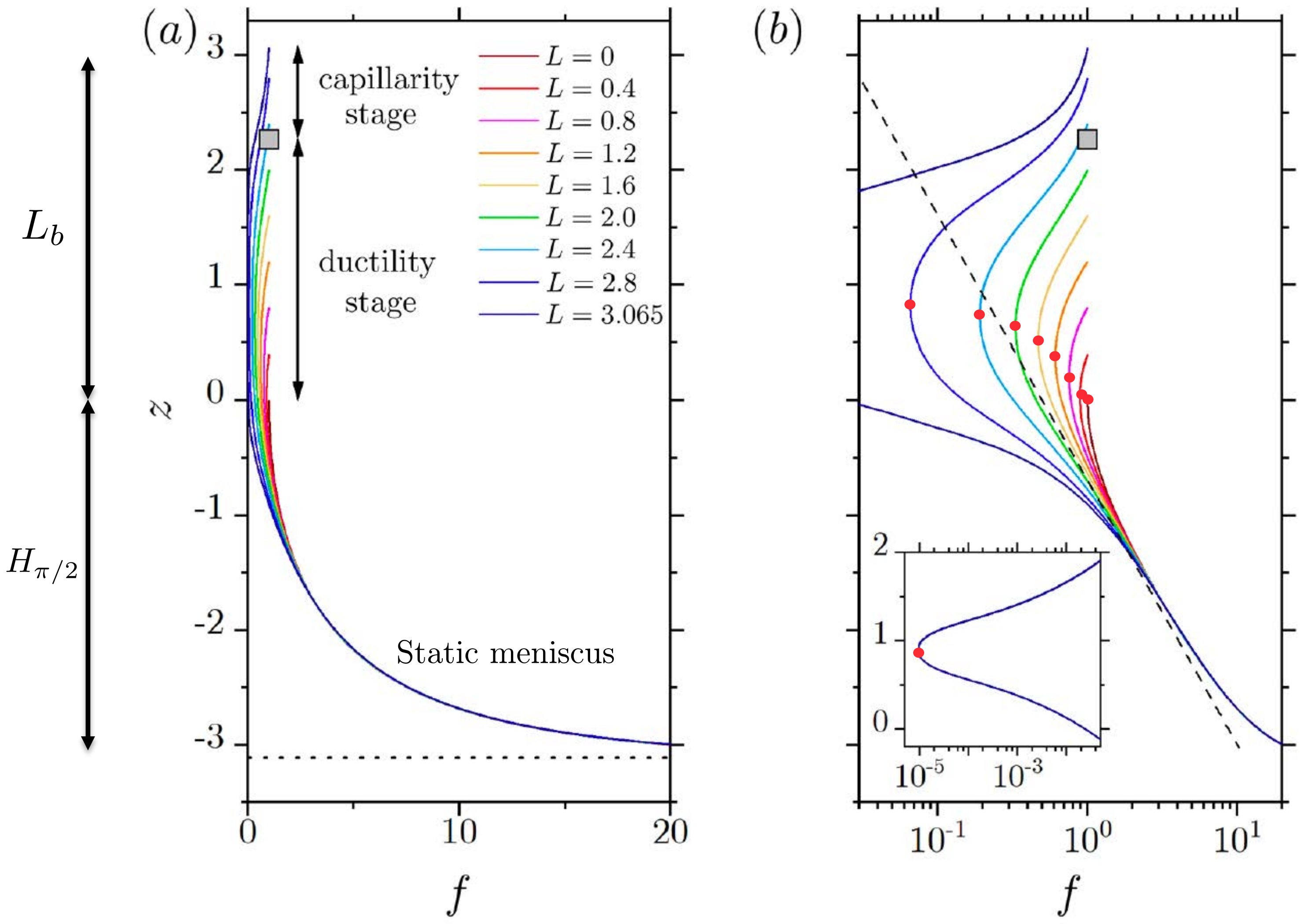
$$\boxed{\varepsilon = \frac{R}{\ell_c}}$$

$$H_s(\varepsilon, \theta) \approx -\sin\theta \ln\varepsilon + \sin\theta \left(n - 2\ln\sin\frac{\theta}{2} \right) \qquad (\varepsilon \lesssim 0.1)$$

$$\boxed{H_{\pi/2}(\varepsilon) \approx \ln\frac{2}{\varepsilon} + n} \qquad n = 0.108.$$

Ligament drawing: dynamics

$$\boxed{H_b(\varepsilon, Ca) = H_{\pi/2}(\varepsilon) + L_b(\varepsilon, Ca)} \qquad (\varepsilon \lesssim 0.1)$$



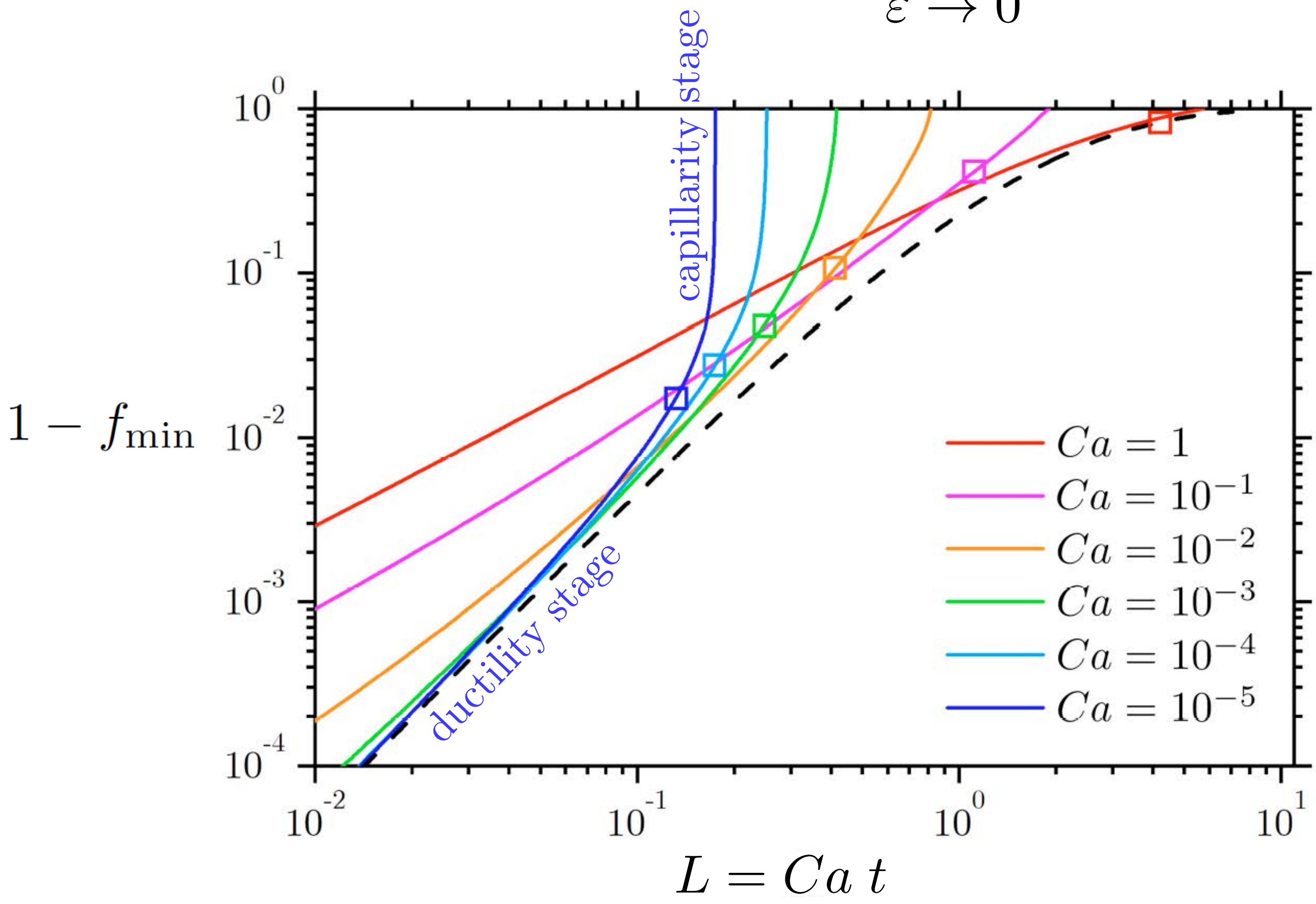
$Ca = 0.3$

$\varepsilon = 0.1$

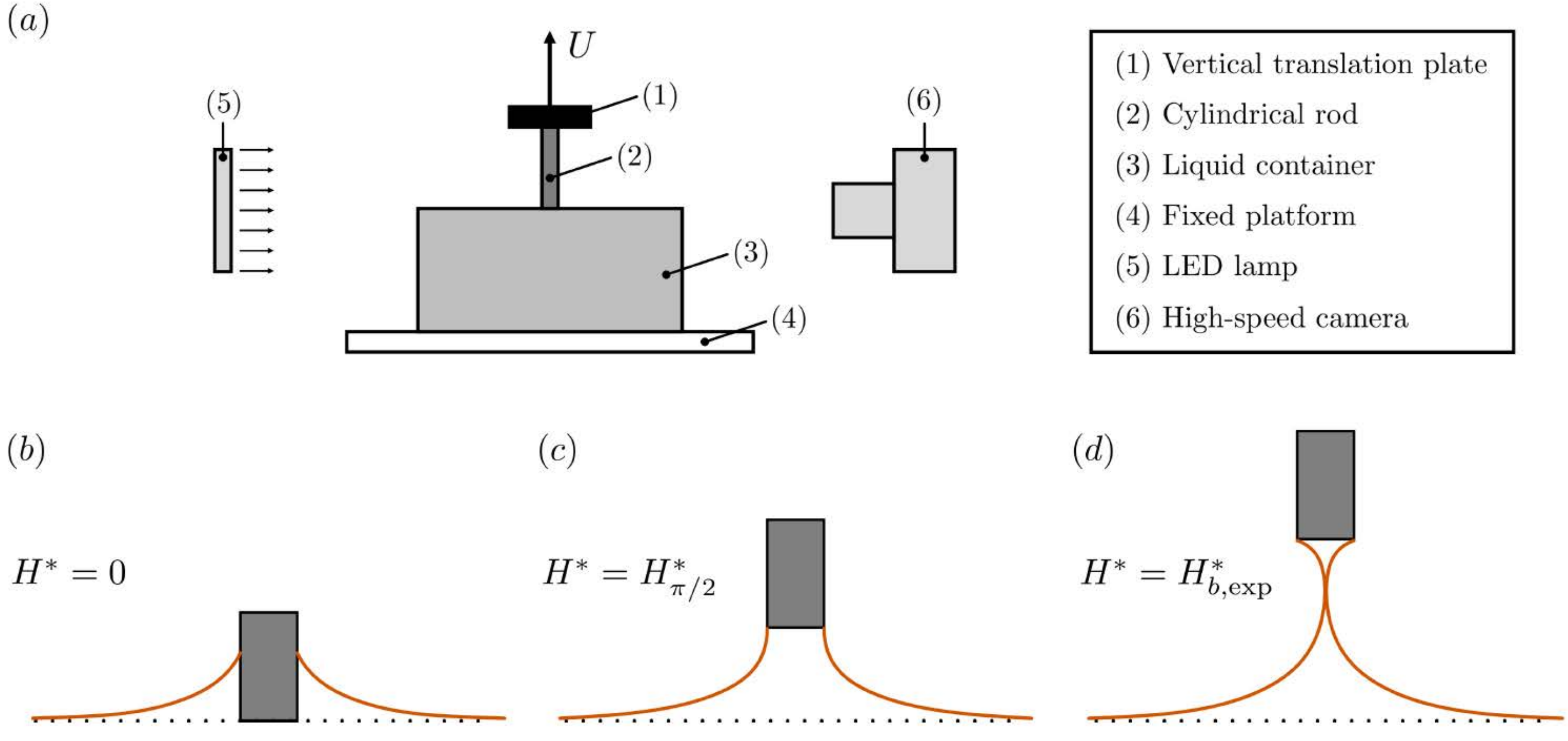
● $f_{\min}$

Ligament stretching dynamics

$\varepsilon \to 0$

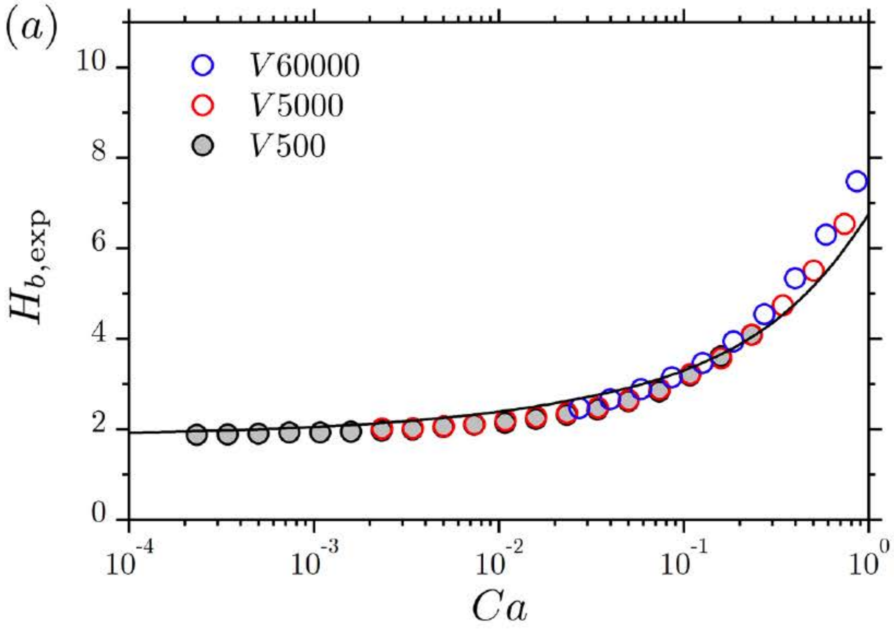


Ligament stretching: experiments

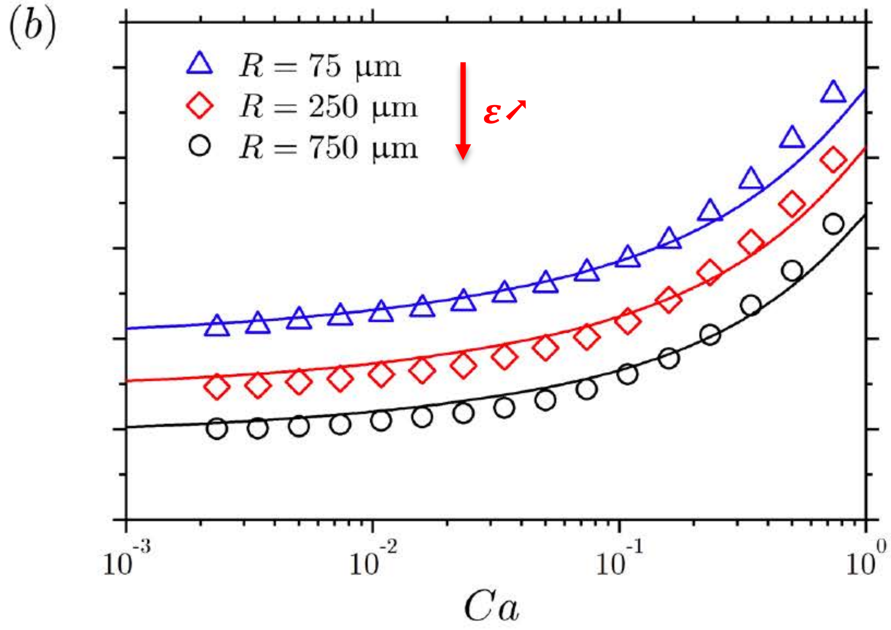


(1) Vertical translation plate
(2) Cylindrical rod
(3) Liquid container
(4) Fixed platform
(5) LED lamp
(6) High-speed camera

Ligament stretching:
Experiments vs 1D modeling



Changing the liquid viscosity

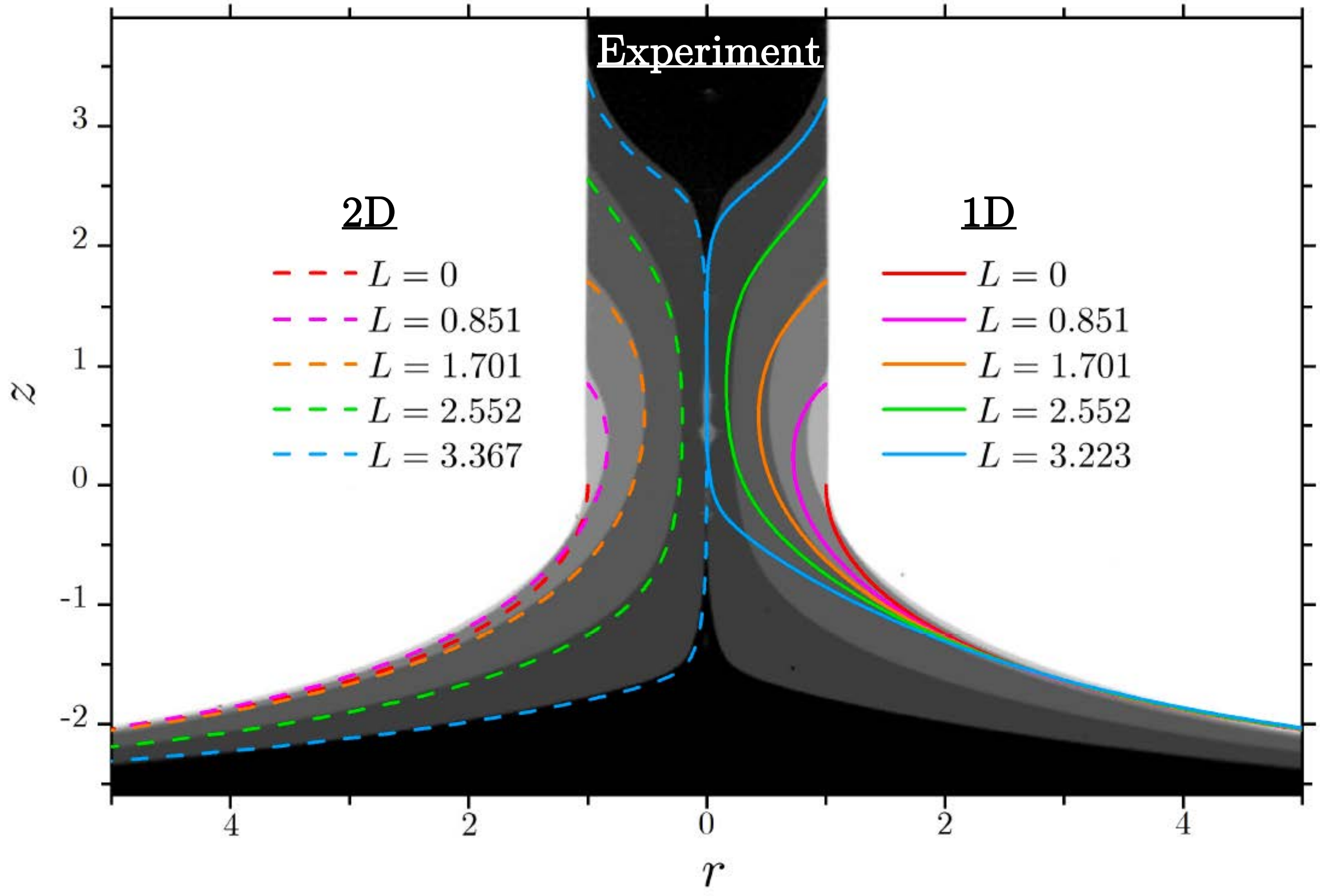


Changing the rod radius R

Ligament stretching: 2D modeling

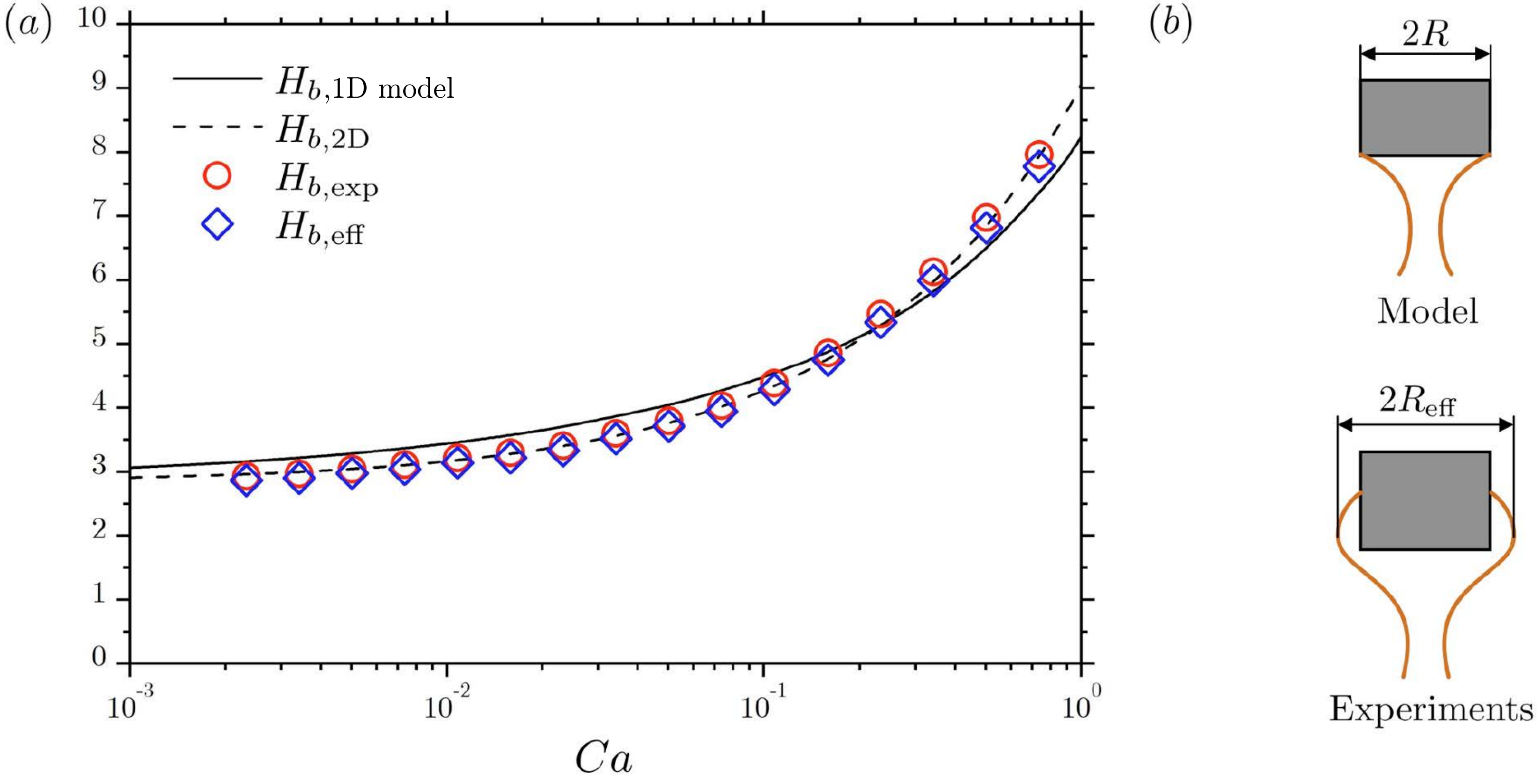
Comsol 2D axisymmetric + ALE for moving mesh with moving boundaries

Ligament stretching: comparison



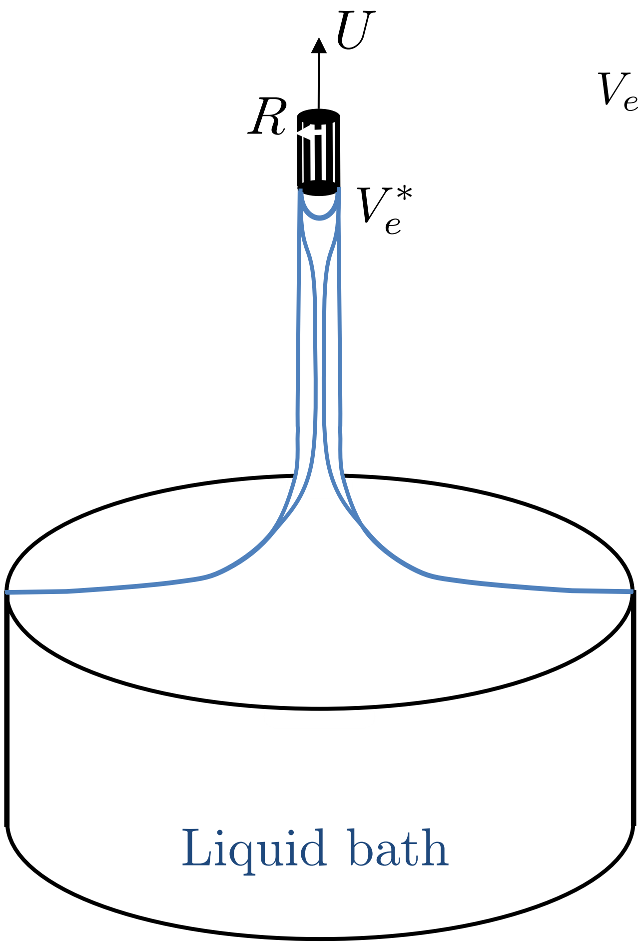
→ Good prediction of the 1D model despite the 2D effects

Ligament stretching: 2D effects ?

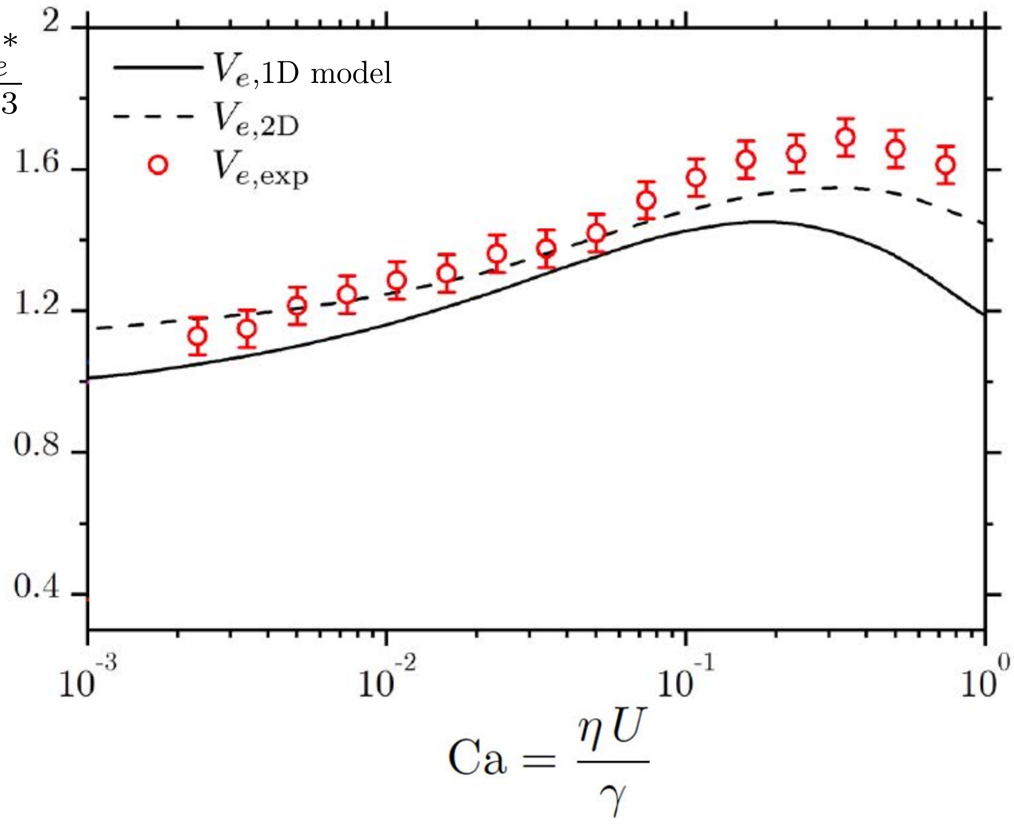


True 2D effects !

Ligament stretching: Volume entrained?



$$V_e = \frac{V_e^*}{R^3}$$



Small variation of the entrained volume !!!

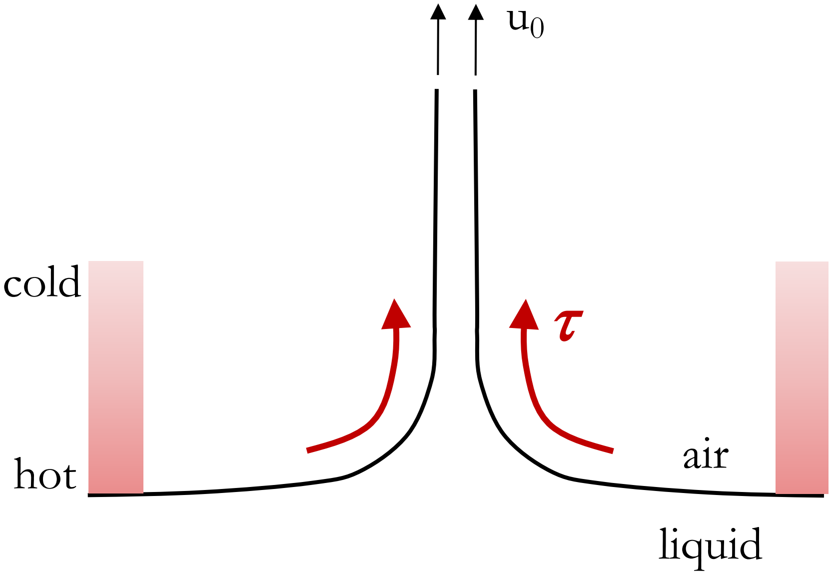
Conclusions

- Unsteady dynamics for vertical stretching of liquid films and ligaments
  - Strong influence of the break-up mechanism: van der Waals *vs.* capillarity
- Good prediction of the 1D, provided Ca < 1 and $\varepsilon \ll 1$
- Small deviation at Ca > 0.1 due to fiber coating effect
- Prediction of break-up heights: quantitative agreement with experiments
  - Always small and maximum for Ca ~ 1 :
    - For films: about 10 times ℓ_c
    - For ligaments: about 10 times R

- How can we pull stable films or ligaments?

Pulling stable liquid films

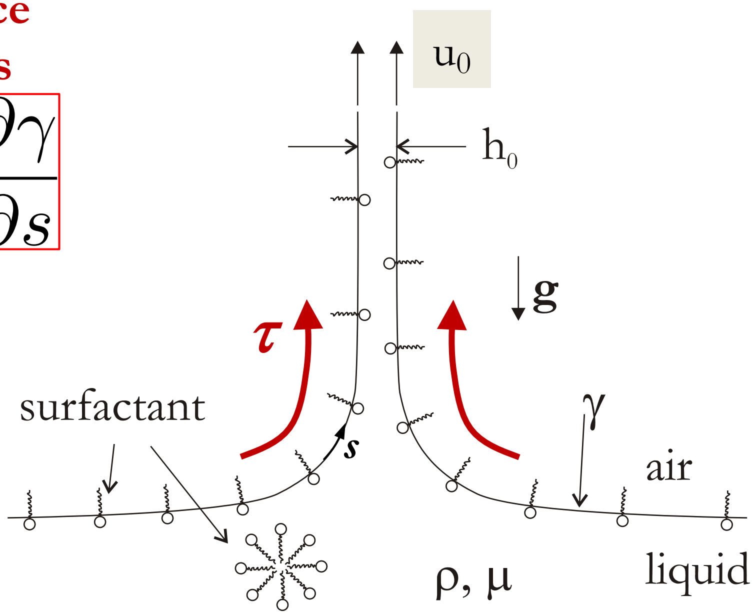
Film of pure material



Surface stress

$$\boxed{\tau = \frac{\partial \gamma}{\partial s}}$$

soap film



Thermocapillary-assisted pulling of contact-free liquid films, Scheid B., van Nierop E. & Stone H. A., Physics of Fluids 24, 032107 (2012)

Surfactant-induced rigidity of interfaces: a unified approach to free and dip-coated films, Champougny L., Scheid B., Restagno F., Vermant J. & Rio E., *Soft Matter* **11,** 2758 (2015)