Droplet breakup - Analyse de la rupture de gouttes: Droplet oscillations in a turbulent flow

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Introduction

Motivation:

- Lamb theory assumes axisymmetric shapes.
- Droplet breakup in turbulent medium due to a resonant mechanism.

Objective:

- Characterize deformation as an expansion in spherical harmonics.
- Oscillations of a droplet immersed turbulent flow will be investigated..

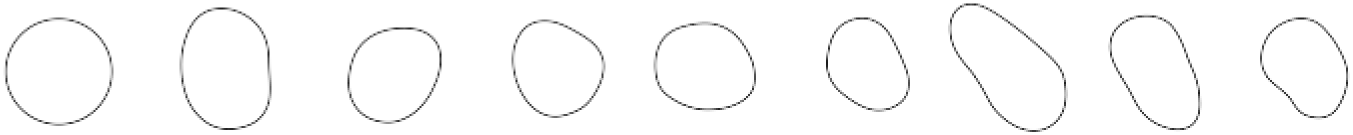


Figure 1: Droplet deformation in a turbulent flow. From left to right the evolution of a single droplet.

[1] H. Lamb, “On the Vibrations of an Elastic Sphere,” Proc. Lond. Math. Soc., vol. s1-13, no. 1, pp. 189212, 1881.
[2] B. Lalanne, O. Masbernat, and F. Risso, “A model for drop and bubble breakup frequency based on turbulence spectra,” AIChE J., vol. 65, no. 1, pp. 347–359, 2019.
[3] Perrard, S., Rivière, A., Mostert, W., & Deike, L. (2021). Bubble deformation by a turbulent flow. Journal of Fluid Mechanics, 920.

Expansion in spherical harmonics

- Any shape that is star-shaped can be described as an expansion of spherical harmonics:

$$R(\theta,\varphi,t) = R_0 \sum_{m=0}^{\infty} \sum_{l=-m}^{m} a_{m,l}(t) Y_m^l(\theta,\varphi)$$

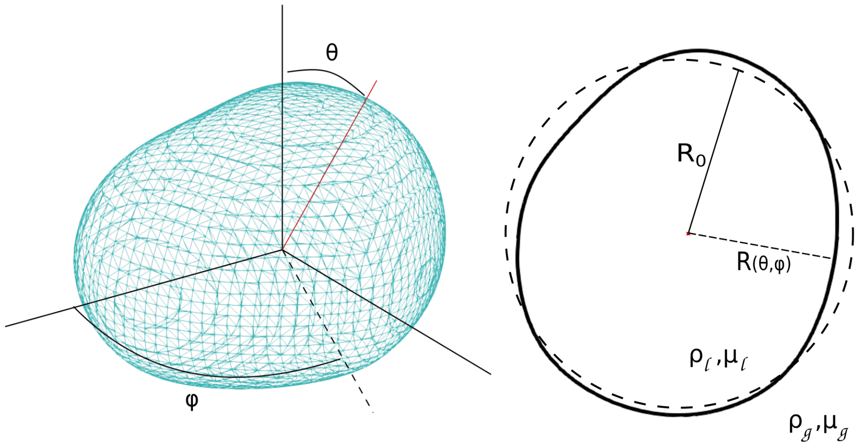


Figure 2. Droplet deformation: sketch and definitions.

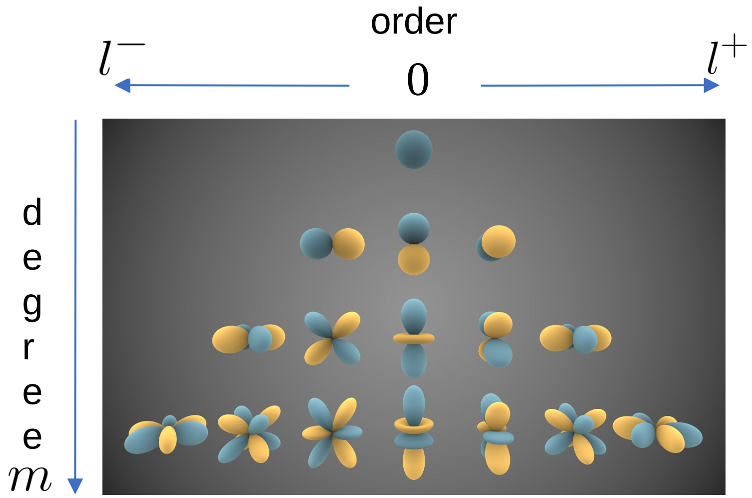


Figure 3. Spherical harmonics representation.

Droplet oscillations in a resting fluid

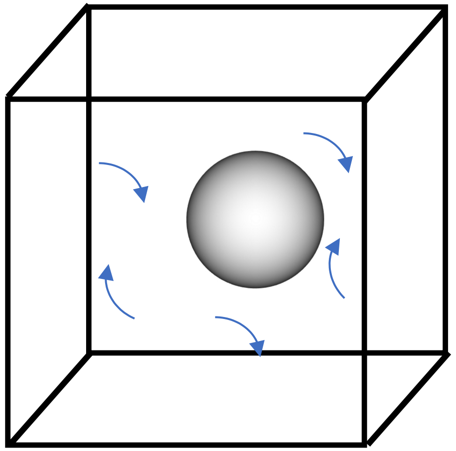
The solution given by [Prosperetti, 1980] is considered. In the limit of weak viscous effects, this equation can be simplified [Lu & Apfel, 1991] as:

$$\omega_m = \omega_{m0}\left[1 - \underbrace{\frac{(2m+1)^2\sqrt{\hat{\rho}\hat{\mu}}}{2\sqrt{2\omega_{m0}\frac{\rho_l}{\mu_l}}R_0[m\hat{\rho}+(m+1)](1+\sqrt{\hat{\rho}\hat{\mu}})}}_{A}\right] \qquad \omega_{m0} = \sqrt{\frac{m(m+1)(m-1)(m+2)}{R_0^3(m\rho_g+(m+1)\rho_l)}}$$

$$\beta_m = \omega_{m0}(A - 2A^2) + \frac{(2+m)[2(m^2-1)+(m+2)\hat{\mu}-(m-1)\hat{\rho}\hat{\mu}+2m(m+2)\hat{\rho}\hat{\mu}^2]}{2(m\hat{\rho}+m+1)[1+\sqrt{\hat{\rho}\hat{\mu}}]^2}\frac{\mu_l}{R_0^2\rho_l}$$

[1] Prosperetti A. Viscous effects on perturbed spherical flows. Q Appl Math (1977) 34(4):339–52.
[2] Lu H-L, Apfel RE. Shape oscillations of drops in the presence of surfactants. J Fluid Mech (1991) 222:351–68.

Droplet database

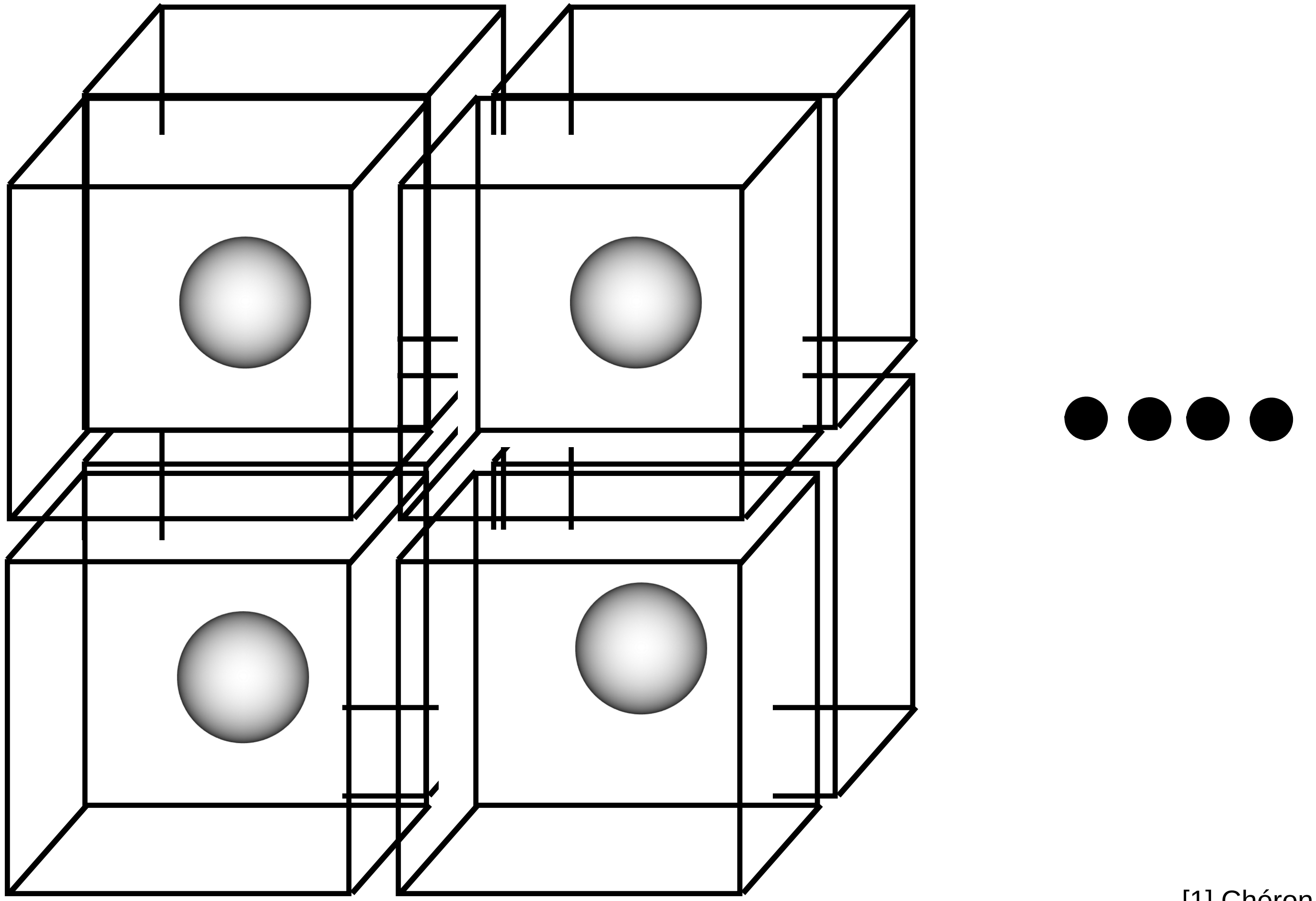


- We consider the liquid-liquid interaction to experience no mixing or mass exchange.
- The Weber number, defined as $We = \frac{\rho_l u'^2 2R_0}{\sigma}$, is set to 0.9 for all cases.
- To produce the turbulent flow, linear forcing adapted to two-phase flows with particles and interfacial flows has been applied with a turbulent Reynolds of

$$Re_\lambda = \frac{\rho_l u' \lambda}{\mu_l} \approx 45$$

- The density and viscosity ratios were both set to $\frac{\rho_g}{\rho_l} = \frac{\mu_g}{\mu_l} = 1$

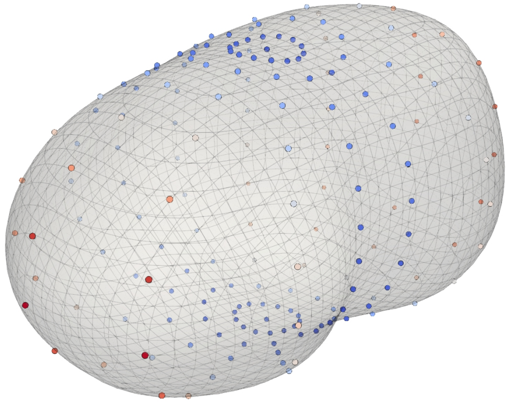
Droplet database



[1] Chéron V, Brändle de Motta JC, Blaisot J-B, Ménard T. Analysis of the effect of the 2D projection on droplet shape parameters. Atomization and Sprays (2022)
[2] Deberne C, Chéron V, Poux A, Brändle de Motta JC. Breakup prediction of oscillating droplets under turbulent flow (2023) Forthcoming.

Expansion in spherical harmonics

- $R(\theta, \varphi)$ is discretized, and the spherical harmonics computed using PySHtools.



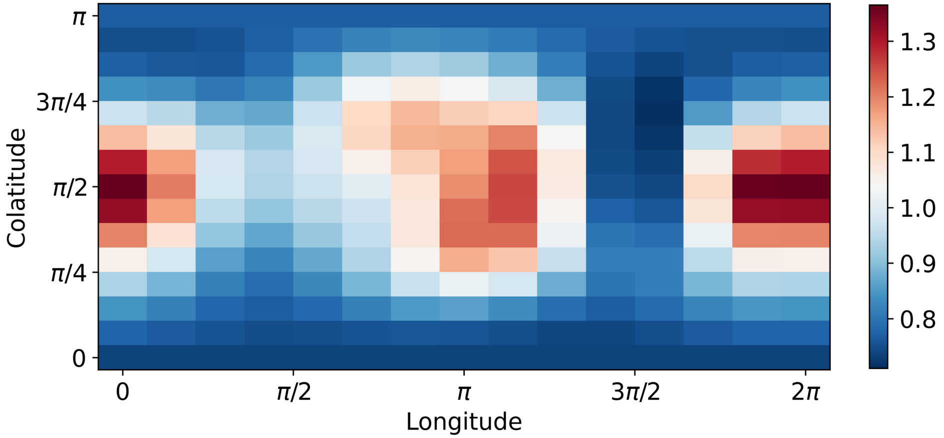
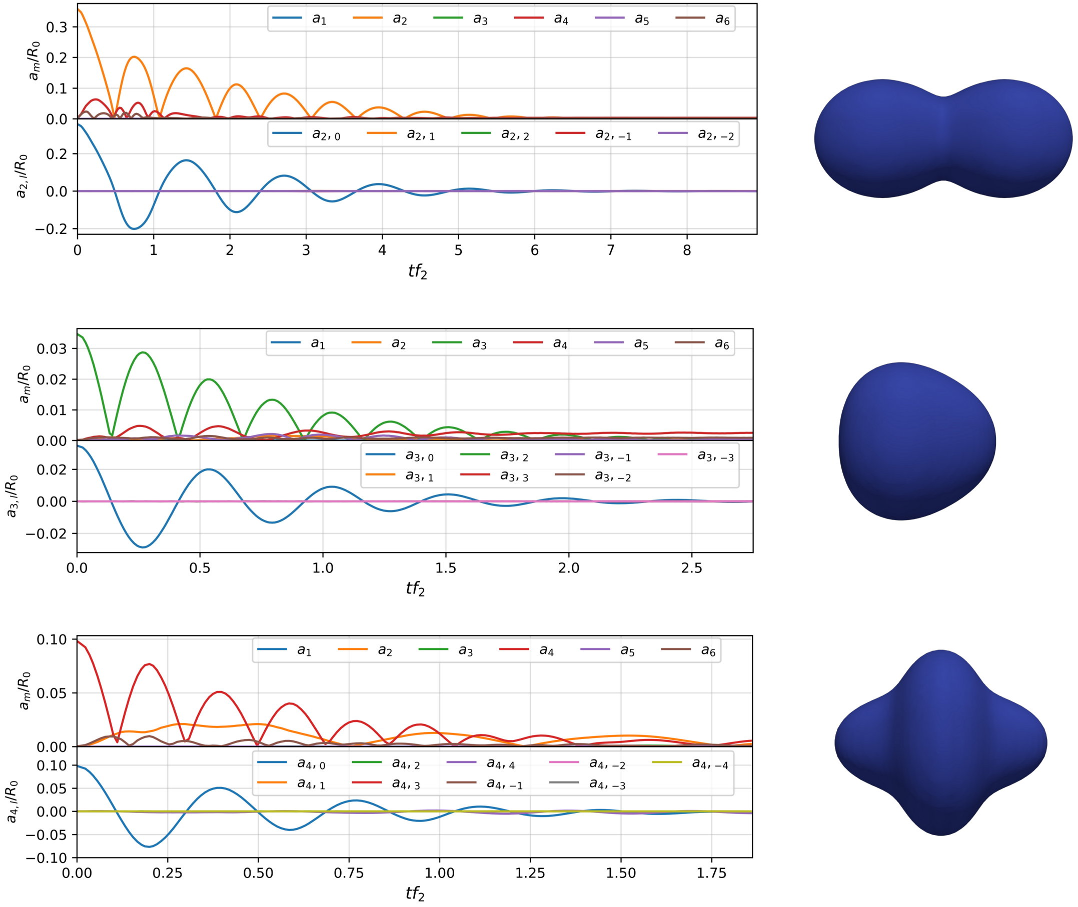


Figure 4. Illustration of the interface mapping of a droplet deformed by a turbulent background flow for a spherical harmonic decomposition of the first 6 modes. (a) The points are distributed around a revolution axis given by the reference frame. (b) The normalized distances between the center and the interface are stored in a grid. The colorbar corresponds in both figures to the values of the function $R(\theta, \varphi)$ normalized by the reference radius R_0.

[1] Wieczorek, M. A., & Meschede, M. (2018). SHTools: Tools for working with spherical harmonics. *Geochemistry, Geophysics, Geosystems*, *19*(8), 2574-2592.

Oscillation of single mode droplets



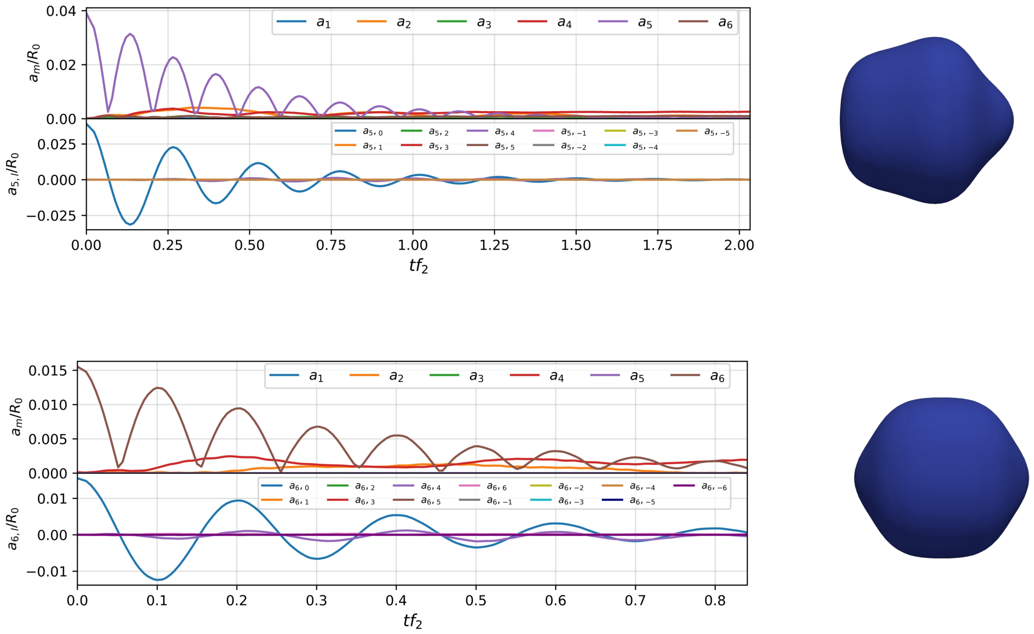


Figure 5. Oscillations of a droplet initialized with a spherical harmonic mode coefficient m = 2–6. (A) $a_{2,0}$ =0.36R_0 , (B) $a_{3,0}$ =0.035R_0 , (C) $a_{4,0}$ =0.1R_0 , (D) $a_{5,0}$ =0.04R_0 , and (E) $a_{6,0}$ =0.016R_0 . The coefficients $a_{m,0}$ were selected in accordance to the highest amplitude of the coefficients found in turbulent flow deformation.

[1] Foote GB. A numerical method for studying liquid drop behavior: Simple oscillation. J Comput Phys (1973) 11(4):507–30
[2] Alonso CT. The dynamics of colliding and oscillating drops. JPL Proc Intern Colloq Drops Bubbles (1974) 1:139–157.

Oscillations of a droplet immersed in a turbulent flow

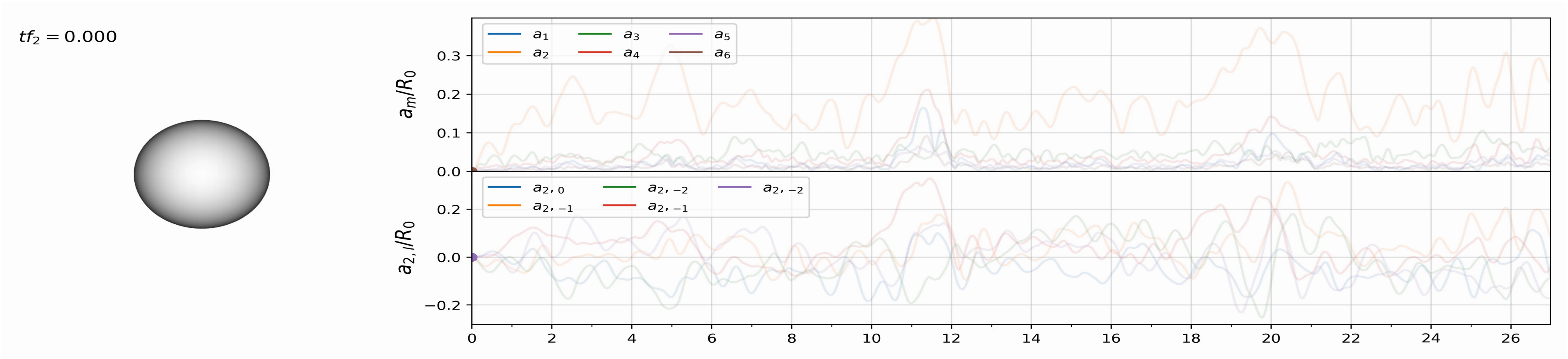


Figure 6. Top image shows the Power spectrum of the spherical harmonic decomposition for modes m = 1 to m = 6 as a function of the dimensionless time tf_2 of one droplet from the database. In the bottom, Coefficients $a_{2,l}$ as a function of the dimensionless time tf_2 of one droplet from the database for l = −2 to l = 2 are shown.

[1] Risso F, Fabre J. Oscillations and breakup of a bubble immersed in a turbulent field. J Fluid Mech (1998) 372:323–55.
[2] Lalanne B, Masbernat O, Risso F. A model for drop and bubble breakup frequency based on turbulence spectra. AIChE J (2019) 65(1):347–59

Initial growth and saturation regime

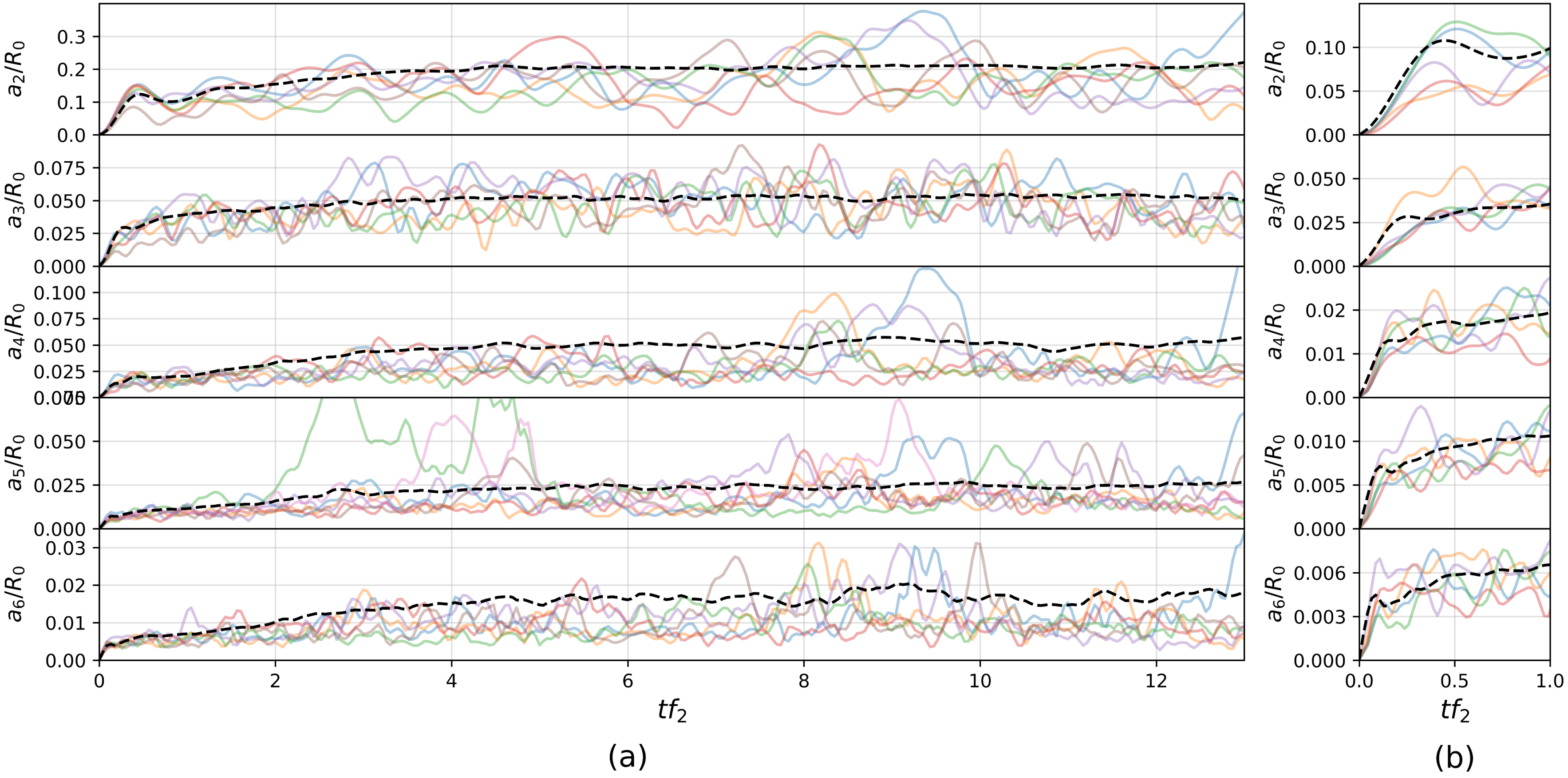


Figure 7. (a) Evolution of the deformation for each individual spherical harmonic mode coefficient am as a function of the dimensionless time tf_2 (individual realizations from the database in solid lines) with its respective ensemble average (dashed line) for We = 0.9. (b) Close up of the initial growth regime

Large deformations and damping factor

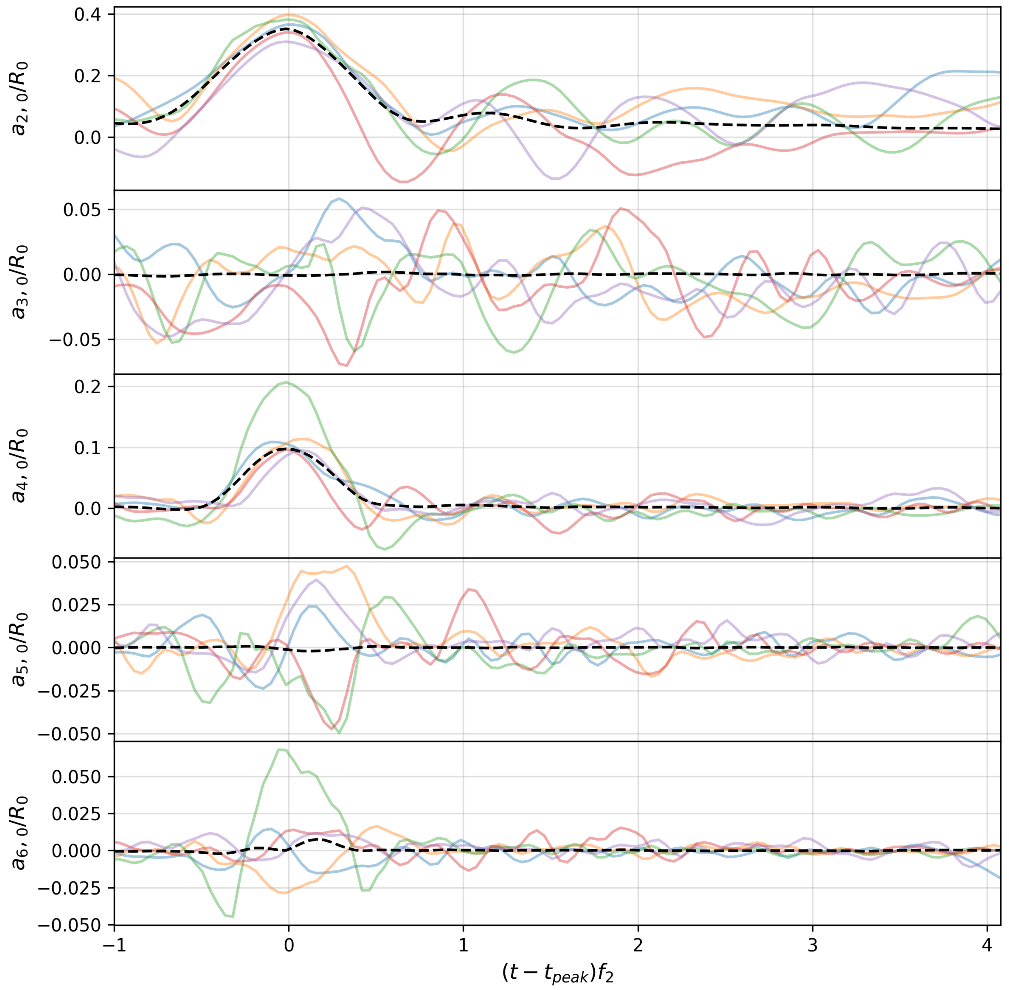


Figure 8. Evolution of the spherical harmonic coefficients $a_{m,0}$ following a high amplitude deformation $a_2 > 0.3$ (individual realizations from the database in solid lines) with its respective ensemble average (dashed line).

[1] Basaran OA. Nonlinear oscillations of viscous liquid drops. J Fluid Mech (1992) 241:169–98.

Large deformations and damping factor

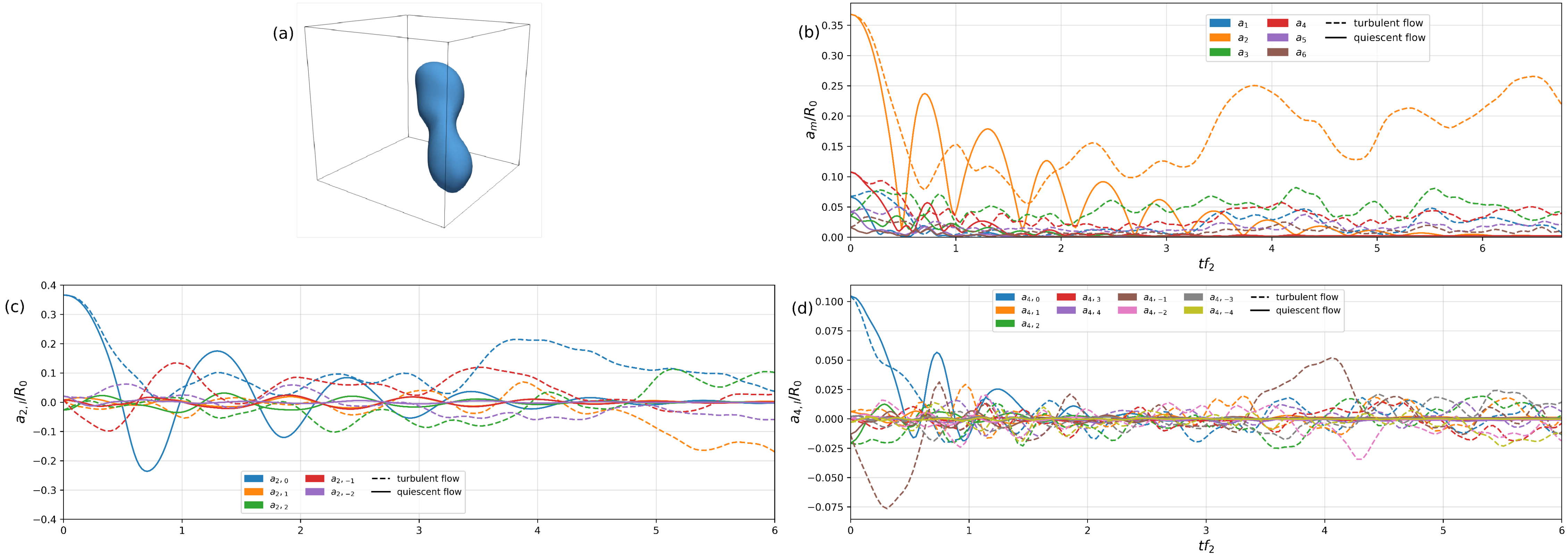


Figure 9. (a) Snapshot of the droplet in its maximum deformation time. (b) $t - t_{peak}$ spectrum coefficients am as a function of the dimensionless time tf_2 of a realization transferred into a quiescent flow. (c) Evolution of the spherical harmonic coefficients $a_{2,l}$ after rotating the reference frame to maximize coefficient $a_{2,0}$. (d) Evolution of the spherical harmonic coefficients $a_{4,l}$ in the same reference frame.

Conclusions

- A droplet database generated by DNS was used to characterize the oscillations of droplets.
- A framework for droplet deformation using a spherical harmonic decomposition of the droplet surface has been presented.
- As a reference case, droplets with single deformation modes are first considered. The oscillatory motion is consistent with the theoretical framework.
- It is shown that mode 2 is dominant in the deformation of a droplet undergoing turbulence-driven deformation.
- Oscillations of each spherical harmonic coefficient are observed.

[1] Roa, I., Renoult, M. C., Dumouchel, C., & Brändle de Motta, J. C. (2023). Droplet oscillations in a turbulent flow. Frontiers in Physics, 11, 1173521.

Conclusions

- The ensemble average of the power spectrum reaches a saturation level related to the droplets natural damping rate and the turbulent cascade.

- The spherical harmonic reference frame is aligned with a large deformation axis, maximizing the $a_{2,0}$ coefficient allowing to observe the coupling between even modes.

[1] Roa, I., Renoult, M. C., Dumouchel, C., & Brändle de Motta, J. C. (2023). Droplet oscillations in a turbulent flow. Frontiers in Physics, 11, 1173521.

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